portional to CM and BC. From C measure the distance CD=this third proportional. D is the required point.

For $BC^2=CM.CD$, or $BC^2=(AB-BC)CD$. $\therefore CD.AB=BC^2+BC.CD$.

$$CD.AB+BC.CD+CD^2=BC^2+2BC.CD+CD^2.$$

$$CD(AB+BC+CD)=(BC+CD)^2.$$

$$AD.CD=BD^2.$$

Also solved by S. A. Corey, J. R. Hitt, F. D. Posey, M. E. Graber, W. W. Landis, and G. W. Greenwood.

249. Proposed by W. W. BEMAN, The University of Michigan.

Given the distances of a point in the plane of a square from three of its vertices, to find the side of the square.

250. Proposed by W. W. BEMAN, The University of Michigan.

Given the distances of a point in the plane of an equilateral triangle from the vertices; to find the side of triangle. [Perkins' *Geometry*, Olney's *Geometry*.]

I. Solution by F. D. POSEY, San Mateo, Calif.

Consider the general case, viz: Given the distances of a point in the plane of a regular n-gon to three consecutive vertices, to find the side of the n-gon.

Let the vertices be A, B, C in order, say clockwise, and P the given point. Let PA, PB, $PC\equiv a$, b, c, respectively. Let $\angle ABP=\alpha$, $\angle PBC=\beta$, taking these clockwise if P be without the angle ABC, and counter-clockwise if P be within. Call the side of the n-gon, x.

There are now two cases: (1) P within the angle ABC of the n-gon, (2) P without this angle. In the first case $\alpha+\beta=\frac{n-2}{n}\pi$. $\therefore \cos\beta=-\cos\frac{2\pi}{n}\cos\alpha+\sin\frac{2\pi}{n}\sin\alpha$. $\therefore \sin\alpha=\operatorname{cosec}\frac{2\pi}{n}\cos\beta+\cot\frac{2\pi}{n}\cos\alpha$.

In the second case $\alpha+\beta=2\pi-\frac{n-2}{n}\pi$. $\therefore \sin\alpha=-\operatorname{cosec}\frac{2\pi}{n}\cos\beta-\cot\frac{2\pi}{n}\cos\alpha$

Now $\cos\alpha=\frac{b^2+x^2-a^2}{2bx}$ (when c is between b and a we have $\cos(2\pi-\alpha)=\cos\alpha$), and $\cos\beta=\frac{b^2+x^2-c^2}{2bx}$ (when a is between b and c we have $\cos(2\pi-\beta)=\cos\beta$). In both cases (1) and (2), $\sin^2\alpha+\cos^2\alpha=(\operatorname{cosec}\frac{2\pi}{n}\cos\beta+\cot\frac{2\pi}{n}\cos\alpha)^2+\cos^2\alpha=1$, which equation after substituting the above values for $\cos\alpha$ and $\cos\beta$ reduces to:

$$\left[\left(\cot\frac{2\pi}{n}+\operatorname{cosec}\frac{2\pi}{n}\right)^2+1\right]x^4+2\left\{\left(\cot\frac{2\pi}{n}+\operatorname{cosec}\frac{2\pi}{n}\right)\left[\cot\frac{2\pi}{n}(b^2-a^2)\right.\right.$$

$$\left.\left.+\operatorname{cosec}\frac{2\pi}{n}(b^2-c^2)\right]-a^2-b^2\right\}x^2+\left[\cot\frac{2\pi}{n}(b^2-a^2)+\operatorname{cosec}\frac{2\pi}{n}(b^2-c^2)\right]^2$$

$$+(b^2-a^2)^2=0.$$